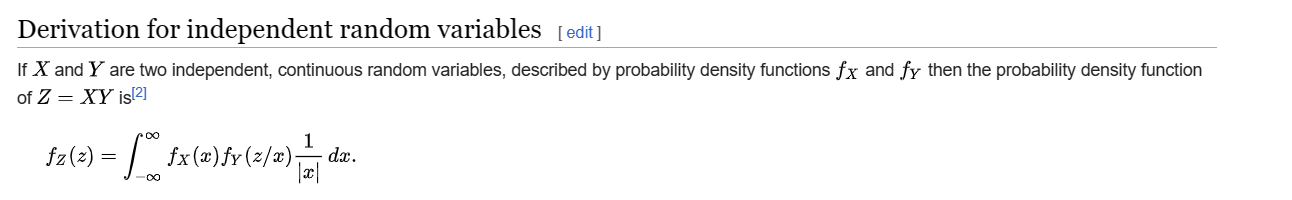
Product distribution

Intro

A distribution that is constructed by product of random variables.

PDF



Proof:

Method 1:

First, we prove CDF of product distribution.

Lastly, we differentiate CDF to get PDF.

For first part, see proof below.

For last part,

=

+

=

=

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Method 2:

Again, first, we prove CDF of product distribution.

Lastly, we differentiate CDF to get PDF.

For first part, see proof below.

By definition of CDF

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=

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where

refers Heaviside step function.

For last part,

=

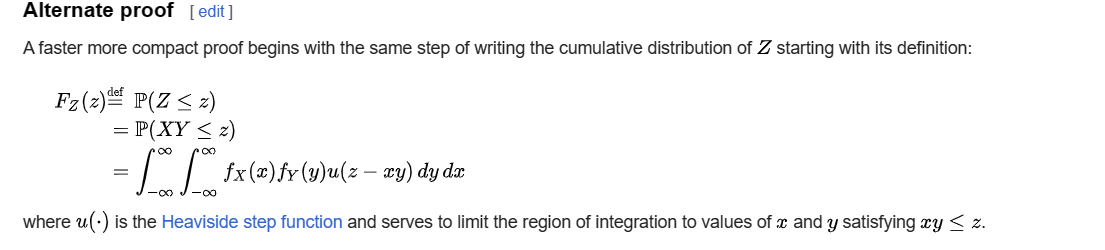
=

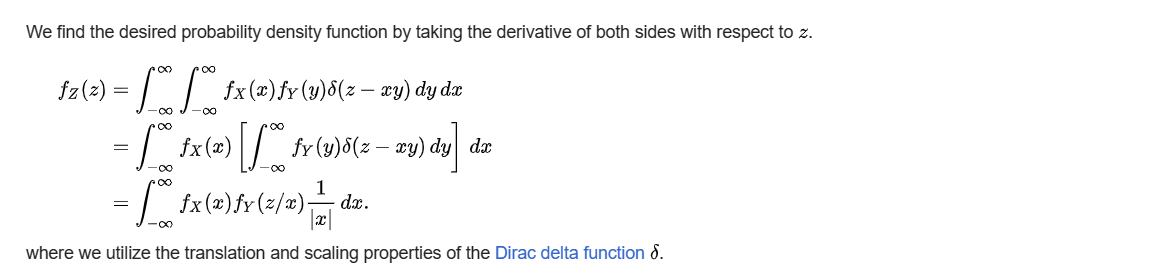
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=

where

DiracDelta refers Dirac Delta function.



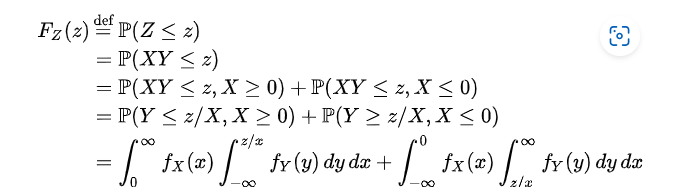


CDF





Proof:



Expected Value

=

Proof:

One can prove it by law of total expectation.

=

(by definition)

=

(by law of total expectation)

=

(since Y is independent to random variable X given by assumption)

=

(since Y is a constant for random variable X given by assumption)

=

(since is always a constant by axiom of expected value)

=

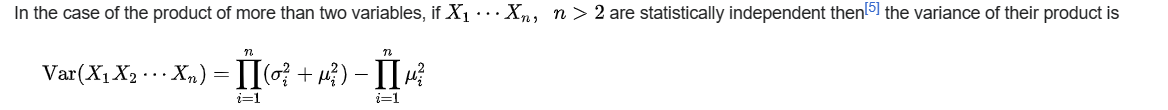
(since Y is independent to random variable X given by assumption)

=

(by mutation of multiplication)

Variance

Generalization



Suppose:

be independent random variables.

And

Then

=

=



Proof:

Proof by induction

Let :

= -

Base step:

is true. Since

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= -

is also true. See the following proof.

By property of variance, we have that

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(since is independent)

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+

(adding and subtracting some terms)

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+

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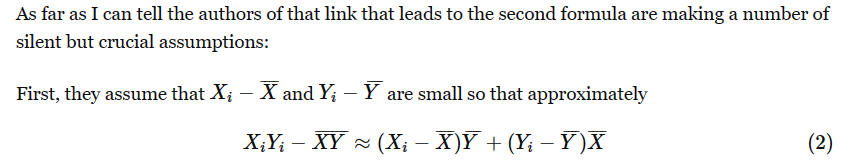
(by property of variance)

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(by property of variance)



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-> 0

-> 0

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On the other hand,

-> 0

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-> 0

Doing so for .

Futhermore,

= -> = 0

And thus,

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= 0

-> 0

Hence,

= 0

-> 0

Adding the above two approximations together. We have that

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-> 0 + 0 = 0

=>

-> 0

Combing the previous two claims, we can get that

=>

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=

=

=

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-> 0

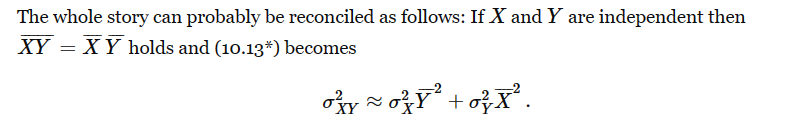
By previous claim,

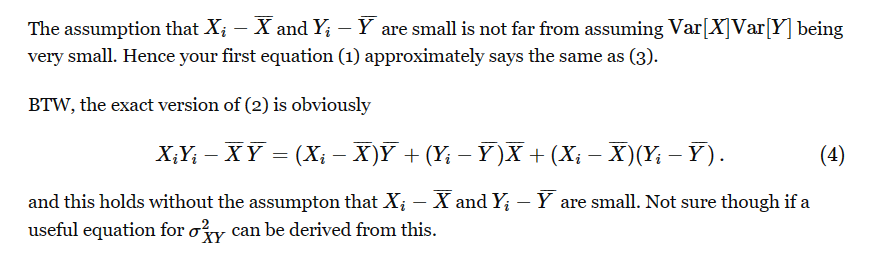
-> 0

And thus,

->

-> 0





From the reply in the following website.

[Variance of product of two random variables ($f(X, Y) = XY$) - Mathematics Stack Exchange](https://math.stackexchange.com/questions/4271183/variance-of-product-of-two-random-variables-fx-y-xy)

Hence, we have completed the proof of

Inductive step:

By inductive hypothesis, it is true for for any and is an integer.

We have to prove that it is also true for .

Explain it with details.

:

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While we have to prove

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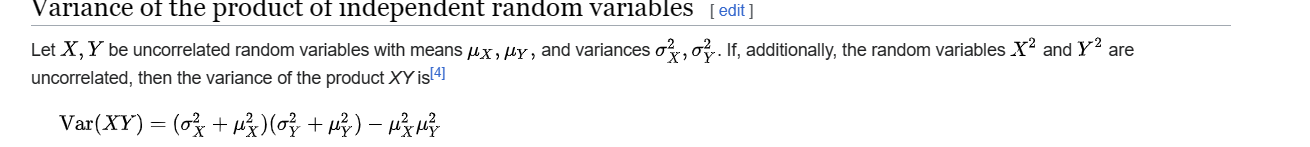
By inductive hypothesis, we have that

=

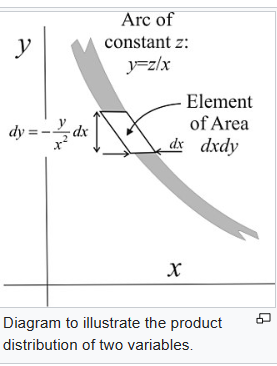
-

Imply this formula to above formula.

Two independent random variables



Diagram



Ref

[Distribution of the product of two random variables - Wikipedia](https://en.wikipedia.org/wiki/Distribution_of_the_product_of_two_random_variables)